Brief information about the project

Name of the project	AP14869472 «Investigation of secular perturbations in multi-planet systems with spherical bodies of variable mass»
Relevance	Observational astronomy testifies that the masses of real celestial bodies are variable. In this connection, the problem of formation and dynamical evolution of planetary systems at the non-stationary stage of their evolution, when mass variability is the leading factor of evolution, is investigated. The idea of the project is to identify the effects of the variability of the masses of planets and of the central star on the dynamic evolution of planetary systems. The isotropic and anisotropic changes in the masses of the bodies included in the system are considered. Body mass decreases due to separating particles and increases due to joining particles, may be reactive forces. This project is aimed at calculating the secular perturbations of the orbital elements of planets, based on aperiodic motion over a quasi-canonical section, in various systems of variables and in different forms of the equation of perturbed motion. The fundamental difference of the project idea from other works and the scientific novelty of the project are the use of the perturbation theory, which based on a special selected aperiodic motion over a quasi-conic section, developed by project participations. for the study of gravitating systems with variable masses.
Purpose	The aim of the project is to identify possible evolutionary tracks of planetary systems, when the leading factor of evolution is the variability of the masses of the planets and of the central star, in cases of isotropic and anisotropic changes in the masses of bodies.
Objectives	In all concepts of the formation of planetary systems, there is a stage of non-stationarity, when the mass of the central star decreases due to radiation and increases due to the matter falling on it from the cosmic environment. Also, at the stage of non-stationarity of planetary systems, the masses of the planets grow due to the fall of substances from the accretion disk. The study of the problem will be conveniently divided into two cases depending on the nature of the change in masses: ➢ Cases of isotropic changes in masses; ➢ Cases of anisotropic changes in masses in the presence of reactive forces. At the same time, it is also advisable to study separately two planetary problems with variable masses and many planetary problems (the number of planets is more than two) with variable masses. Following research objectives are set:

	1. Derivation of the motion differential equations of many spherically symmetric bodies with variable masses, with a point-by-point description of these bodies dynamics, in a relative coordinate system with the origin in the center of the parent star. 2. Define systems of osculating elements - variables to describe specific problems and corresponding equations of perturbation theory. 3. Derivation of perturbing functions to use perturbation theory in the form of the Lagrange equation. 4. Derivation of perturbing functions for the canonical perturbation theory equation in the analogues of the second Poincaré system of variables. 5. Derivation perturbing functions for using the perturbation theory in the form of the perturbed motion equation in the Newton's form. 6. Decomposition and correction of perturbing functions into series, with any required degree of accuracy, on the elements of aperiodic motion along a quasi-conical section, by methods of computer algebra in Mathematica analytical calculations. 7. Derivation of evolutionary equations, by averaging over mean longitude, in different variables and in different forms. 8. Solve numerically the derived equations of secular perturbations and visualize the results. 9. Analyze the obtained results and describe the possible evolutionary tracks of multi-planet systems with variable masses. 10. An overall analysis of the project work and identification of further promising problems in the dynamics of planetary systems with variable masses. Writing the final report.
Expected and achieved results	The results obtained in the course of the project determine the possible evolutionary paths of multi-planet systems with variable masses. The obtained scientific results are of particular interest in the formation of the worldview of earthlings in the Universe about life in general and as one of the civilizations developed on planet Earth. The expected results lying at the intersection of mathematics, celestial mechanics, astrophysics will be of interest to the community of natural sciences. These results will be disseminated to potential users, the scientific community and the general public, including bachelors, masters and PhD students of the al-Farabi Kazakh National University. The new scientific results obtained will be applied to the study of exoplanetary systems. The target consumers of new results are astronomers, astrophysicists and scientists dealing with non-stationary problems of theoretical astronomy, celestial mechanics, as well as for

	doctoral students, undergraduates and senior university students. The results obtained in the team of authors are at the world level, and the expected results will determine the world level in this field of science. The Republic of Kazakhstan is actively developing the space industry, the Baikonur Cosmodrome is located in our republic, space science will develop at a faster pace. This explains the social demand and economic interest in the implementation of the project and obtaining its results.
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List of publications with links to them	• ***Published articles included in the list recommended by CQASEME RK:*** 1. M. Minglibayev, A. Kosherbayeva. Evolution equations of multi-planet systems with variable masses // Journal of Mathematics, Mechanics and Computer Science. -2022, V. 116, No. 4, 35-45. https://doi.org/10.26577/JMMCS.2022.v116.i4.04 2. M. Minglibayev, A. Kosherbayeva. System of linear differential equations of secular perturbations of exoplanets with variable masses // “Bulletin of the National Engineering Academy of the Republic of Kazakhstan” and “Computing Technologies Federal Research Center for Information and Computational Technologies”. -2022, No. 3 (1), 134-146. • ***Published an article in the journal included in the SCOPUS database:*** Chichurin A., Prokopenya A., Minglibayev M., Kosherbayeva A. Symbolic-Numeric Computation in Modeling the Dynamics of the Many-Body System TRAPPIST // Lecture Notes in Computer Science Lecture Notes in Computer Science. 2023.V.14075. P.469-482. doi: 10.1007/978-3-031-36024-4_36 • ***Published abstracts in the proceedings of international conferences:***

	1. Minglibayev M.Zh., Prokopenya A.N., Kosherbayeva A.B. Investigation of the dynamic evolution of planetary systems with isotropically varying masses Complex Planetary Systems II. Kavli-IAU Symposium 382. University of Namur, Belgium, July 3-7, 2023. -P.42-43. 2. Chichurin A., Prokopenya A., Minglibayev M. and Kosherbayeva A. Symbolic-Numeric Computation in Modeling the Dynamics of the Many-Body System TRAPPIST. The International Conference on Computational Sciences. Prague, Czech Republic, 3-5 July, 2023, On-line. -P.99. 3. Prokopenya A., Minglibayev M., Ibraimova A. Derivation of the evolution equations in the restricted three-body problem with variable masses by using Computer Algebra. Applications of Computer Algebra – ACA 2023, Warsaw, Poland, July 17 – 21, 2023. P.68. 4. Minglibayev M., Prokopenya A., Kosherbayeva A.B. The problem of many bodies with isotropically varying masses. Applications of Computer Algebra – ACA 2023, Warsaw, Poland, July 17 – 21, 2023. - P. 70.
Patents	-